A Simple Universal Cellular Automaton and It's One-way and Totalistic Version

Jürgen Albert
Karel Culik II

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Jürgen Albert

Lehrstuhl für Informatik II
Universität Würzburg
D-8700 Würzburg, Germany

Karel Culik II

Department of Computer Science
University of Waterloo
Waterloo, Ontario N2L 3G1 Canada

ABSTRACT

In this paper we first derive two normal form constructions for cellular automata to transform any given (one-dimensional) cellular automaton into one which is one-way and/or totalistic. An encoding of this restricted type of automaton together with any initial configuration becomes the input of our small universal cellular automaton, using only 14 states. This improves well-known results obtained by simulation of small universal Turing machines and also some recent results on universal totalistic cellular automata.

1. Introduction

Interest in cellular automata (CA) has been renewed since their application to the study of complex systems [19-21]. In this context the universality of CA was discussed in [21] and open problems about universal CA with a small number of states were stated in [22]. We give some results here.

A cellular automaton (CA) is said to be **universal** if it can simulate every Turing Machine or, even stronger, if it can simulate every CA of the same dimension. The first universal cellular automaton was given in the famous work of J. von Neumann on the simulation of self-reproduction. He gave a 2-dimensional universal and self-reproducing CA with 29 states. This was improved to 20 states in [1].

Also well known is the work on small universal Turing Machines (cf. [11, 12, 15]). In this case the goal is to minimize simultaneously the number of states and the number of tape symbols. In [12] it is shown that there is a universal

Turing Machine with either 4 symbols and 7 states or with 6 symbols and 6 states. Smith [16] has shown that any Turing Machine with m symbols and n states can be simulated by a one-dimensional CA whose uniform cell has $m+2n$ states. Thus either one of the universal TM from [12] yields a universal CA with $4+2.7 = 6+2.6 = 18$ states. Here we improve this result and show that there is a universal one-dimensional CA with 14 states. We believe that some more effort in the detailed "low-level programming" of our basic strategy can decrease this by 2 or 3 states. However, the minimal number of states of a universal one-dimensional CA could still be much smaller, since already CA with 3 or 4 states show astonishingly complex behaviour.

The well known "Game of Life" is a "semi-totalistic" CA which means that the next state of a cell only depends on its own current state and the sum of the states of its neighbors. S. Wolfram also introduced an even more restricted type of CA, called totalistic. The next state of a cell of such CA depends only on the sum of all the states in its neighborhood, including its own. D. Gordon [9] has shown that totalistic one-dimensional CA can simulate every Turing Machine and we will strengthen this and show that every one-dimensional CA can be simulated by a totalistic one. We will use this result in the construction of our small universal CA. We will also use the result that every one-dimensional CA can be simulated by a one-way (unidirectional) one-dimensional CA [6,8,18] and that one-way CA are equivalent to trellis automata [4].

2. Preliminaries

As we will consider only one-dimensional and homogeneous cellular automata in this paper we will restrict ourselves to this special case in the following definitions. The more general terminology can be found in [5] or [17].

Intuitively, a cellular automaton consists of a doubly infinite array of cells. All cells are identical copies of one single finite automaton. The local transition function of each cell only depends on the actual states of its left and right neighbor and itself. Thus, a computation of a cellular automaton can be defined in the straightforward way as the synchronous application of the local transition function at each cell. The set of states always contains a so-called quiescent state q with the property: if a cell and its left and right neighbors are quiescent at time t then this cell is quiescent at time t+1. As we assume finite input for the cellular automaton this implies that there is a finite number of nonquiescent cells in the initial configuration and in every subsequent configuration as well.

Definition 2.1 A (one-dimensional, homogeneous) cellular automaton is a triple $A = (Q, d, q)$, where Q is a finite set of states, d is the local transition function, $d: Q \times Q \times Q \rightarrow Q$ where the arguments of d are used in the following meaning d(state of left neighbor, own state, state of right neighbor), and q in Q is the quiescent state, i.e. it holds $d(q, q, q) = q$.

A configuration of A is a mapping $C : Z \rightarrow Q$, where Z denotes the set of integer numbers such that $C(i) = q$ (quiescent state) for all but finitely many i's. The set of nonquiescent cells of a configuration C is called the support of C.

A computation of A now consists of a sequence of configurations $C_0, C_1, C_2, \ldots, C_n, \ldots$, where C_0 is the initial configuration and each configuration C_{i+1} is generated by simultaneous invocation of the transition function d for all cells of A with states as given by C_i.

3. Simulation by one-way automata

For our construction of a universal cellular automaton in subsequent chapters we will need an automaton which is totalistic and one-way.

A cellular automaton A with set of states Q and transition function $d : Q \times Q \times Q \rightarrow Q$ is called one-way, if d only depends on the state of the own cell and the state of its right neighbor cell. Thus we can write the transition function of a one-way cellular automaton in the form $e : Q \times Q \rightarrow Q$ with the convention that the arguments of e consist of the states of the own cell and those of its right neighbor.

In this section we will outline a transformation of an arbitrary cellular automaton to a one-way cellular automaton in order to specify bounds for the increase in the number of states and in time-steps needed in the simulation.

In [18] a similar technique was used for the case of real- time cellular automata, more general cases were considered in [6] and [8].

The construction of the one-way automaton can be described briefly by:

1. merging of each state with the state of its right neighbor
2. shifting to the left during the state transition.

Let $s_1, s_2, \ldots, s_k$ be states in a configuration, such that $s_1 \neq q$ and let $t_0, t_1, \ldots, t_{k+1}$ be the states after one transition as shown below

$$\begin{array}{cccccccccccc}
q & q & q & s_1 & s_2 & s_3 & \cdots & s_k & q & q & q \\
q & q & t_0 & t_1 & t_2 & t_3 & \cdots & t_k & t_{t+1} & q & q
\end{array}$$

Then it takes 2 time-steps in our simulating one-way automaton to produce the transition

$$\begin{array}{cccccccccccc}
q & q & q & s_1 & s_2 & s_3 & \cdots & s_k & q & q & q \\
q & t_0 & t_1 & t_2 & t_3 & & \cdots & t_{k+1} & q & q & q
\end{array}$$

For the given cellular automaton A let Q be the set of states and $d : Q \times Q \times Q \rightarrow Q$ the transition function.

As defined above, in the simulating one-way cellular automaton A' the transition function d' will consider only the states of its own cell and its right neighbor cell.

Let $Q' = Q \cup Q{\times}Q$ and define $d' : Q'{\times}Q' \rightarrow Q'$ as follows

$$d'(u,v) = uv \quad \text{for} \quad (u,v) \neq (q,q)$$

$$d'(q,q) = q$$

$$d'(uv,\ vw) = d(u,v,w)$$

$$d'(q,qu) = d(q,q,u)$$

$$d'(wq,q) = d(w,q,q)$$

for all u,v,w in Q.

Thus, the transition shown above is completed in two time-steps by A' in the following way

$$\begin{array}{ccccccccc}
q & q & s_1 & s_2 & s_3 \cdots & s_k & q & q & q \\
 & & & & & & & & \\
q & qs_1 & s_1\,s_2 & s_2\,s_3 & \cdots & s_k\,q & q & q & q \\
 & & & & & & & & \\
t_0 & t_1 & t_2 & & \cdots\ t_k & t_{k+1} & q & q & q
\end{array}$$

The details of this construction are straightforward, so we can state the following theorem:

Theorem 3.1 For every cellular automaton A with k states there exists a one-way cellular automaton A' which simulates A twice slower and A' needs at most $k^2 + k$ states.

In the above construction of the simulating one-way cellular automaton one can delay the given transitions by "aging" the states in Q (eg. a, a', a'', a''',...) to obtain slower expansion of the nonquiescent cells.

Therefore it is clear that the following corollary holds:

Corollary 3.2 For every cellular automaton A with k states and for every $m{\geq}1$ there exists a one-way cellular automaton A' which simulates $A(m{+}1)$-times slower and A' needs at most $k^2 + mk$ states.

Later, we will have to apply this corollary for $m{=}3$ in the construction of a universal one-way cellular automaton.

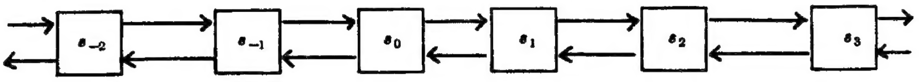
4. Simulation by totalistic automata

In this section we will show that each cellular automaton can be simulated (without loss of time) by a cellular automaton which has a totalistic transition function and uses up to four times as many states as the original automaton.

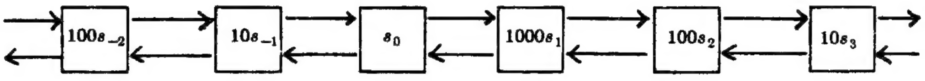
Definition 4.1 A cellular automaton A with set of states Q and transition function $f : Q \times Q \times Q \rightarrow Q$ is called totalistic, if $Q \subseteq N$ (non-negative) and there exists a function $f' : N \rightarrow N$ such that $f(x,y,z) = f'(x+y+z)$ for all x,y,z in Q.

The transformation of an arbitrary cellular automaton whose states are identified with non-negative integers is now accomplished by a cyclic "coloring" of cells. We use four different powers of a basis such that in the number representation of the sum of three neighboring states left neighbor, right neighbor and own cell are still identifiable by the position of a missing entry.

Consider e.g. a cellular automaton A with set of states $Q = \{1,2,...,n\}$ and transition function $d : Q \times Q \times Q \rightarrow Q$ and let the figure below depict part of a configuration of A.



Let $B = n+1$ be the basis for the colouring factors 10, 100 and 1000 (in B-ary notation). Then the above configuration changes to the following:



Thus our new set of states is

$$Q' = \{sm \mid s \in Q,\ m \in \{1,10,100,1000\}\} .$$

So Q' contains $4n$ states and we can define now the (partial) totalistic transition function $d' : N \rightarrow Q'$ as follows

$$d'(xyz) = 10d(x,y,z)$$

$$d'(xyz0) = 100d(x,y,z)$$

$$d'(yzx) = 1000d(x,y,z)$$

$$d'(z0xy) = d(x,y,z)$$

for all x,y,z in Q.

Again xyz, $xyz0$, $yz0x$, $z0xy$, 10, 100, 1000 are to be interpreted as numbers in the B-ary system.

It is now straightforward, that the cellular automaton A' with set of states Q' and totalistic transition function d' correctly simulates A without loss of time and it therefore holds

Theorem 4.2 For every cellular automaton A there exists a totalistic cellular automaton A' which simulates A without loss of time and has at most four times as many states as A.

If only one-way cellular automata are considered, the above construction can be simplified using only three different powers of B for the colouring of the cells, whereby the one-way property of the given automaton is preserved.

Corollary 4.3 For every one-way cellular automaton A there exists a one-way totalistic cellular automaton A' which simulates A without loss of time and has at most three times as many states as A.

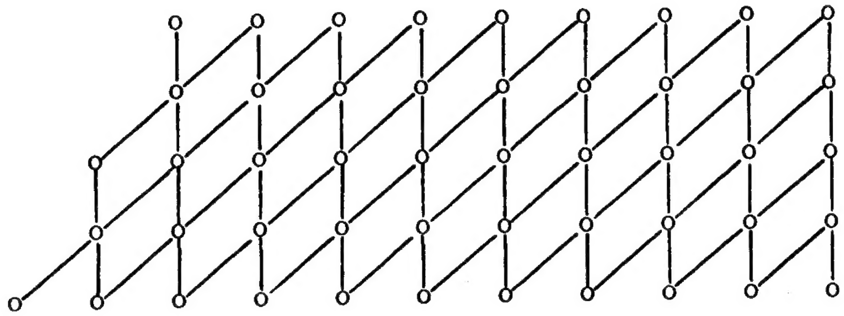
For the more general case of an arbitrary (regular) systolic network this technique of coloring its underlying graph is used in [23] to show that actually every systolic network can be transformed into a totalistic one.

5. Informal description of construction

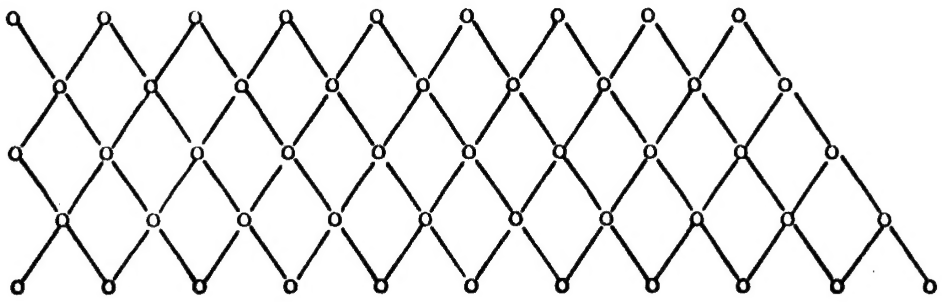
In the previous section we have shown that any cellular automaton can be simulated by one which is one-way and totalistic and such that the expansion of the nonquiescent part to the left can be delayed by any constant factor. We choose $m=3$ to achieve expansion to the left at most at half speed.

Our universal cellular automaton U will simulate any given one-way totalistic cellular automaton A (which expands at most at half speed to the left) with any given initial configuration. We will encode this pair as an initial configuration of the universal cellular automaton U. A constant number (depending only on the number of states of A) of steps of U is needed to simulate one step of A.

The time-space diagram (unrolling) of the given one-way automaton then looks like



This unrolling is obviously equivalent to the following

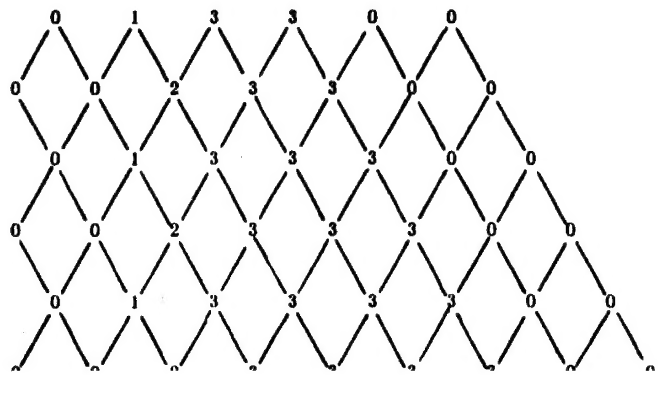


which will be the basis of our construction of the universal cellular automaton U. Note that the computations which took place in one cell of the first automaton therefore being depicted in vertical lines in the first figure, are now shifting to the right in each step.

Example: Let A be a totalistic one-way cellular automaton with set of states Q = {0,1,2,3,4} and transition function d given by

i	0	1	2	3	4	5	6
$d(i)$	0	0	1	3	2	3	3

Since A is one-way the nonquiescent states expand to the right only. An example of a computation of A follows.



In the simulation of the automaton A each of the nonquiescent cells of A will be encoded as a block of constant size of cells in U. This block contains the given transition-table for the cells of A, and by the marking of the appropriate table-entry also the encoding of the current state of a cell, i.e. if a transition $d(i+j) = k$ takes place in a cell of A, in the corresponding block the $(i+j)$th table-entry - which holds an encoding of the number k-becomes marked.

So in our automaton U the main actions to simulate one time-step of the given automaton A will be grouped into three phases

phase 1: send information about current state to left neighbor block

phase 2: send information about current state to right neighbor block

phase 3: process information to look up next state.

6. Outline of construction

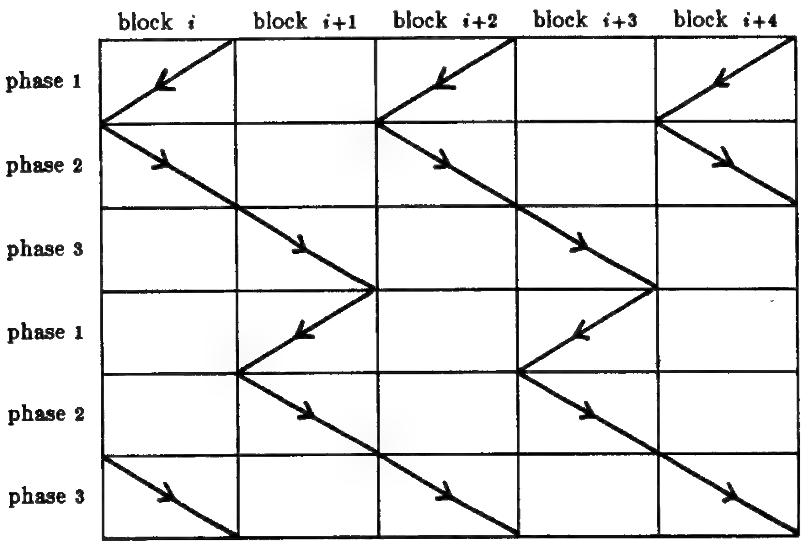
As shown above the next states are always computed between the two cells whose states are to be considered for the table-lookup. Therefore it is convenient to use alternatively "activated" blocks and "empty" blocks and to change the roles of both in the odd and even steps (beats) of the computation. This is achieved by sending messenger signals over these blocks where the signals have to complete various tasks during the three phases mentioned above.

phase 1: start from the right end of an active block and find the table-entry representing the current state; activate a number of signals according to the current state which are moving to the empty block to the left; continue to move to the left end of the block.

phase 2: bounce at the left end of the block and find once more the table-entry representing the current state; accordingly activate signals which are now moving to the empty block to the right; while clearing all temporary marks continue to move to the right end of the block, leaving an empty block behind.

phase 3: cross over to the block to the right; while moving to the right end of this block process the signals which have been accumulated here during the previous phases; bounce at the right end.

The flowing of messenger signals over some adjacent blocks during the phases of two consecutive beats can be depicted as follows:



Furthermore we have to take care that after each beat a new block at the right-hand end of the nonquiescent part can be activated to simulate a possible expansion to the right. This is achieved by copying (at least) one complete block to the right during each beat and by activating one new block which starts in the encoding of the quiescent state.

By our construction we can assume that there is no expansion to the left and so we only need to activate the leftmost block at each second beat. This is realized by a special block and messenger signals.

Details of these constructions and encodings, the transition-table for U and examples are found in the appendix.

Theorem 6.1: There exists a universal cellular automaton U with 14 states which can simulate any given totalistic one-way cellular automaton A with any initial configuration.

In [16] it is shown (Theorem 4) that any Turing machine T with m tape-symbols and n states can be simulated by a cellular automaton with $m+2n$ states. Thus either of the universal Turing machines with 6 symbols and 6 states or with 4 symbols and 7 states (cf. [12]) yields a universal cellular automaton with 18 states.

We can now apply the normal form constructions of our previous sections to the universal cellular automaton U of Theorem 6.1 to generate universal cellular automata which are also one-way and/ or totalistic.

Corollary 6.2: For every universal cellular automaton U with n states there exists a universal one-way cellular automaton U_1 with $n^2 + n$ states.

Corollary 6.3: For every universal cellular automaton U with n states there exists a universal totalistic cellular automaton U_2 with $4n$ states.

Corollary 6.4: For every universal cellular automaton U with n states there exists a universal one-way totalistic cellular automaton U_3 with $3(n^2 + n)$ states.

7. Appendix

For the encoding of the given transition-table as a block of cells in U we can assume w.l.o.g. that for the given totalistic one-way cellular automaton A the set of states Q is defined as $Q = \{3,4,5,...,k\}$, where 3 is identified with the quiescent state in A.

Each block consists now of a sequence of $2k+1$ sub-blocks of the form $\#z\underbrace{000...000}_{i-times}\underbrace{111...111}_{j-times}$ and a block-endmarker #, where z is in $\{0, 1\}$, $i+j = 2k+1$ and if in the given transition function d we have $d(m)=n$ then in the mth sub-block of each block it holds $j=n$. For all other sub-blocks we have $2j=3$. Thus a block consists of $(2k+2)^2$ cells.

If in a cell of the original one-way totalistic automaton a transition $d(r)=s$ takes place then in the corresponding block in U exactly in the first r sub-blocks from the left this z equals 1 and in all other sub-blocks $z=0$.

The computation of the next transition in the simulating automaton U then takes $3(2k+2)^2 + 7(2k+1)$ time steps which constitute one "beat" of U.

Example: Let $Q = \{3,4,5\}$ be the set of states and the transition-function d be defined as

i	6	7	8	9	10
$d(i)$	3	4	4	5	4

Then a transition by $d(9)=5$ corresponds to the following situation in the block at the beginning of the beat:

```
#100000000111#100000000111#100000000111#100000000111#100000000111#100000000111
#100000001111#100000001111#100000011111#000000001111#0000000001aa#
```

Here aa at the end of the block shows that this block is activated and that the new state 5 has to be sent to left and right neigbors.

It can also be seen easily that the transition-function has been extended to

i	1	2	3	4	5	6	7	8	9	10	11
$d(i)$	3	3	3	3	3	3	4	4	5	4	3

An inactive, "empty" block then looks like

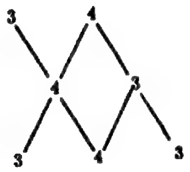
#000000000111#000000000111#000000000111#000000000111#000000000111#0000000000111
#000000001111#000000001111#000000011111#000000001111#000000000111#

If a block is activated at the beginning of a beat, his right neighbor block will become activated at the next beat.

In order to be able to expand, it is therefore necessary to start a new activation from the left end at each second beat. This is done simply by sending a synchronizing signal up and down over a sufficiently long sequence of 1's. The first block after this synchronizing left part contains only table-entries representing the quiescent state. So each second beat the "quiescent block" at the left end is activated and no expansion to the left can occur.

At the right end copying of a block is performed by pushing an encoded form of a block to the right and simultaneously leaving an empty block behind, which will be activated later as representing the original quiescent state. During one beat a little more than a complete block is created at the right, but since activation of blocks is initiated from the left - synchronized with the beat - this does not affect the simulation of the nonquiescent part of the original totalistic one-way automaton.

Example: The following three snapshots show the simulation of two transitions



where the transition-table is given as

i	1	2	3	4	5	6	7	8	9
$d(i)$	3	3	3	3	4	3	4	4	3

In this case one beat of U consists of 363 single transition-steps. The following are the initial configuration of U, the configuration after the first beat (363 steps), and after the second beat (726 steps).

```
qqqq111111111111111111111111111111111111111111111111111111111111111111111111111111111111111111111111
111111111111111111111111111a111111111111111111111111111111111111111111111111111111111111111111111111
1111111111111111111111111111111111111111111111111111111111111111111111111111111111111111111111111111
11111111111111111111111111111111111111111111111111111111111111111#
1#00011111111#00000111#0000000111#0000000111#0000000111#0000000111#0000000111#0000000111#0000000111#
#1000000111#1000000111#1000000111#1000000111#1000000111#1000000111#0000000111#0000000111#00000001aa#
#0000000111#0000000111#0000000111#0000000111#0000001111#0000000111#0000001111#0000001111#0000000111#
#1000000111#1000000111#1000000111#1000000111#1000001111#0000000111#0000001111#0000001111#00000001aa#
#1000000111#1000000111#1000000111#1000000111#1000001111#1000000111#1000001111#1000001111#1000000111#
#1000000111#1000000fg1g0h0h0h0g1g1h0hhhhggg1h0h0g1g1g0hh0h0hgggghhh0h0g1g1g0h0h0h1gg1qqqqqqqqqqq
qqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqq

qqqq111111111111111111111111111111111111111111111111111111111111111111111111111111111111111111111111
1111111111111111111111111111111111111111111111111111111111111111111111111111111111111111111111111111
11111111111111111111111111111111111111111111g1111111111111111111111111111111111111111111111111111111
11111111111111111111111111111111111111111111111111111111111111111#
1#00011111111#00000111#0000000111#1000000111#1000000111#1000000111#1000000111#1000000111#1000000111#
#0000000111#0000000111#0000000111#0000000111#0000000111#0000000111#0000000111#0000000111#0000000111#
#1000000111#1000000111#1000000111#1000000111#1000001111#1000000111#1000001111#0000001111#00000001aa#
#0000000111#0000000111#0000000111#0000000111#0000001111#0000000111#0000001111#0000001111#0000000111#
#1000000111#1000000111#1000000111#1000000111#0000001111#0000000111#0000001111#0000001111#00000001aa#
#1000000111#1000000111#1000000111#1000000111#1000001111#1000000111#1000001111#1000001111#1000000111#
#1000000111#1000000111#1000000111#bjch0hbg1gghhhhhh1g1g1h0h0h0h1g1ghhhhhhg1g1g0h0h0h1g1g1hhhhhhhg1g1
h1h0jqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqq

qqqq111111111111111111111111111111111111111111111111111111111111111111111111111111111111111111111111
111111111111111111111111111a111111111111111111111111111111111111111111111111111111111111111111111111
1111111111111111111111111111111111111111111111111111111111111111111111111111111111111111111111111111
11111111111111111111111111111111111111111111111111111111111111111#
1#00011111111#00000111#0000000111#0000000111#0000000111#0000000111#0000000111#0000000111#0000000111#
#1000000111#1000000111#1000000111#1000000111#1000000111#1000000111#0000000111#0000000111#00000001aa#
#0000000111#0000000111#0000000111#0000000111#0000001111#0000000111#0000001111#0000001111#0000000111#
#1000000111#1000000111#1000000111#1000000111#1000001111#1000000111#1000001111#0000001111#00000001aa#
#0000000111#0000000111#0000000111#0000000111#0000001111#0000000111#0000001111#0000001111#0000000111#
#1000000111#1000000111#1000000111#0000000111#0000001111#0000000111#0000001111#0000001111#00000001aa#
#1000000111#1000000111#1000000111#1000000111#1000001111#1000000111#1000001111#1000001111#1000000111#
#1000000111#1000000111#1000000111#1000000111#10000011qgghhhh0h0g1g1h0h0h0ggggghhh0h0g1g1g0h0h0h0gggg
hghh0h0h1g1g0h0h0h0gg1qqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqqq
```

The following table of state transitions for our universal cellular automaton U will need some explanations. In the transition-table for U the upper row shows the state of the considered cell and the first two columns display the states of the left and right neighbor cells. Blank table-entries mean that these combinations never occur during a simulation, thus any of the states could be entered instead of the blank.

	#	0	1	a	b	c	d	e	f	g	h	i	j	q
#			1				1			h	1			
0	#	0	1	f	1	0	b		1	b	0	#	j	
1	#					#						#	q	
a	#					#								
b	#	b	a	1	d				a		0		j	
c	#	g											0	
d						d								
e	a		a											
f	c							g						
g	#			b	#									j
h	#	0	1	e	h					0	1	#	j	
i			1			c								
j	#	a		b	1									
0		0			0	0		0		1	0		j	
1		0	1	1	0	0	1	0	1	1	0		j	
a		b	a	a	b					1	0			
b		b			b						0		j	
c		c											0	
d		c	d											1
f		0	1											
g		0	1	g	h				q	1	0		0	q
h		0			h					1	0		j	
j		0												
q		0	1							1	0		0	q
#	#		1	1		#		g	e	h	1	#		
0	#	0	1		0			0		1	0	#	j	
1	#	0	1	1		#	1		1	1	0	i		
a	#		a	a		#				1				
b	#	b			b	c				a		#		
c	c		a	a		#			g	1	1	#		
d	c		d						d		1	i		1
e	c		a							1				
f	c		1							g				
g	#		1	g		i			q	1	0	#		q
h	#	0	1		h				q	1	0	#	i	j
i	#		1	g			1		g	1	h	#		
j	#	0	1							1	0			#
q	q		1							1	0		0	q
#	#		1	1						e				
0	#	0			e	#				i				
1	#		1	1						e				
a			a							a				
b		b									f			
c	c		a							a				
e		b												
f	c									g		c		
g	#													
h													j	
#						#								
0		0			0					g	b			
1		0	1	1	0					a	b			
a		b	a							a	b			
b		b									b			
c													0	
e		b												
f		0												
#			d											
0	#	0	1	f	0					1	b	d		

		#	0	1	a	b	c	d	e	f	g	h	i	j	q
c	1	#	0								1	0			
c	a											0			
c	b											0			
c	d						g								
c	f	c									f				
c	g			1											
c	h							#							
c	i			1											
d	#			h											
d	0	d	0									h	#		
d	g			g							g	h			
d	h	i		g							g	h			
e	#	i													
e	0	c	e	g				i		f			i		
e	1		e	f											
f	#	c		f									i		
f	0	#	0			0		i					c		
f	1			f			e				g				
f	g										g	h			
f	h										g	h			
g	#	#		g	1				f	b	f	h			
g	0	d	h	g			#			0	g	h			
g	1	c	h	g	1		#		i		g	h			
g	c	d				a				g		h	i		
g	d								f	e		h			
g	e										g				
g	g		h	g	g						g	h	i		
g	h	#	h	g			i				g	h	i		
g	i	#		g					g				i		
g	j		h	g							g				
g	q		h	g							g	h		1	j
h	#	i											#		
h	0	i	h			0		i			g	h	#		
h	1	i	h	g	1	0	i				g	h	#		
h	c	i													
h	g		h	g	g	h					g	h			
h	h	i	h	g		h					g	h			
h	i	i													
h	j		h	g							g	h			
h	q		h	g							g	h		1	j
i	#							h							
i	0	i	h	c			0			1	g	h	i		
i	1	i													
i	d											c			
i	g						j								
i	h	i	h	0		h	j				g	h	i		
j	0											c			
j	1										f	c			
j	b											c			
j	g		h	g			j								
j	h		h	g		h	j				j	0			
j	q														q
q	0										g				
q	1			1							1				q
q	a			g											
q	g			1				f			d	0		#	q
q	h							f			d	0			
q	q														q

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